On Consensus Measures in Fuzzy Group Decision Making

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Abstract. In group decision making problems, a natural question in the consensus process is how to measure the closeness among experts' opinions in order to obtain the consensus level. To do so, different approaches have been proposed. For instance, several authors have introduced hard consensus measures varying between 0 (no consensus or partial consensus) and 1 (full consensus or complete agreement). However, consensus as a full and unanimous agreement is far from being achieved in real situations. So, in practice, a more realistic approach is to use softer consensus measures, which assess the consensus degree in a more flexible way. The aim of this paper is to identify the different existing approaches to compute soft consensus measures in fuzzy group decision making problems. Additionally, we analyze their advantages and drawbacks and study the future trends.

Keywords: group decision making, consensus process, soft consensus measures.

1 Introduction

In a classical Group Decision Making (GDM) situation there is a problem to solve, a solution set of possible alternatives, and a group of two or more experts, who express their opinions about this solution set of alternatives. These problems consist in multiple individuals interacting to reach a decision. Each expert may have unique motivations or goals and may approach the decision process from a different angle, but have a common interest in reaching eventual agreement on selecting the "best" option(s) [5,8,24]. To do this, experts have to express their preferences by means of a set of evaluations over a set of alternatives.

In a GDM problem, there are two processes to apply before obtaining a final solution [9,13,14,15,18,22,23]: *the consensus process* and *the selection process* (see Figure 1). The former consists in how to obtain the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives. Normally, the consensus process is guided by a human figure called moderator [6,9,22] who is a person that does not participate in the discussion but knows

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the agreement in each moment of the consensus process and is in charge of supervising and addressing the consensus process toward success, i.e., to achieve the maximum possible agreement and to reduce the number of experts outside of the consensus in each new consensus round. The latter refers to how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts. Clearly, it is preferable that the set of experts achieves a great agreement among their opinions before applying the selection process.

At the beginning of every GDM problem, the set of experts have diverging opinions, then, the consensus process is applied, and in each step, the degree of existing consensus among experts' opinions is measured. If the consensus degree is lower than a specified threshold, the moderator would urge experts to discuss their opinions further in an effort to bring them closer. Otherwise, the moderator would apply the selection process in order to obtain the final consensus solution to the GDM problem. In such a way, a GDM problem may be defined as a dynamic and iterative process, in which the experts, via the exchange of information and rational arguments, agree to update their opinions until they become sufficiently similar, and then, the solution alternative(s) is/are obtained. In this paper, we focus on the consensus process.

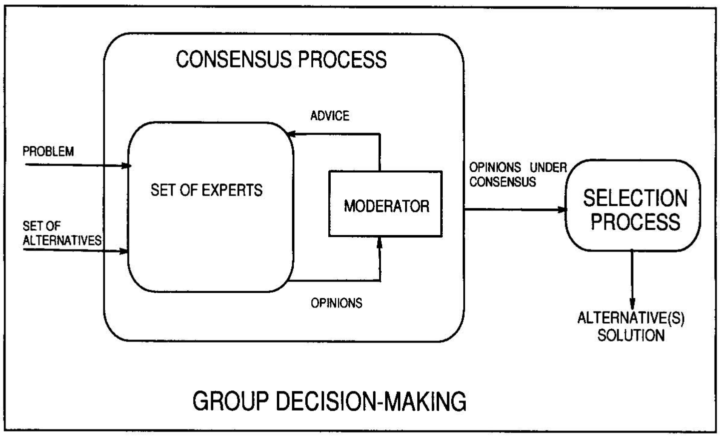

Fig. 1. Resolution process of a GDM problem

A natural question in the consensus process is how to measure the closeness among experts' opinions in order to obtain the consensus level. To do so, different approaches have been proposed. For instance, several authors have introduced *hard consensus measures* varying between 0 (no consensus or partial consensus) and 1 (full consensus o complete agreement) [2,3,26,27]. Thus, using *hard consensus measures*, in [2,3], a distance from consensus as a difference between some average preference matrix and one of several possible consensus preference matrices is determined. In [26] some measures of attitudinal similarity between individuals that is an extension of the classical Tanimoto coefficient are derived.

And, in [27], a consensus measure based on a-cuts of the respective individual fuzzy preference matrices is derived. However, consensus as a full and unanimous agreement is far from being achieved in real situations, and even if it is, in such a situation, the consensus reaching process could be unacceptably costly. So, in practice, a more realistic approach is to use *softer consensus measures* [19,20,21], which assess the consensus degree in a more flexible way, and therefore reflect the large spectrum of possible partial agreements, and guide the consensus process until widespread agreement (not always full) is achieved among experts. The soft consensus measures are based on the concept of coincidence [11], measured by means of similarity criteria defined among experts' opinions.

The aim of this paper is to identify the different existing approaches in the literature to compute soft consensus measures in fuzzy GDM problems and analyze their advantages and drawbacks. To do so, firstly, we identify three different coincidence criteria to compute soft consensus measures: *strict coincidence among preferences*, *soft coincidence among preferences* and *coincidence among solutions*. Then, we analyze their application in consensus processes of fuzzy GDM problems and study their drawbacks and advantages. Furthermore, we describe the new advanced approaches, which use the above coincidence criteria, allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible and adapt the consensus process to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round.

In order to do this, the paper is set up as follows. In Section 2, we present the different approaches proposed in the literature to obtain soft consensus measures in fuzzy GDM problems. In Section 3, we discuss their advantages and drawbacks. The new advanced approaches are shown in Section 4. Finally, some concluding remarks are pointed out in Section 5.

2 Approaches to Obtain Soft Consensus Measures in Fuzzy GDM Problems

In this section, we analyze different existing approaches in the literature to obtain soft consensus measures in a fuzzy GDM problem.

As aforementioned, soft consensus measures are based on the coincidence concept [11], i.e., measuring the existing coincidence among expert's opinions by means of similarity criteria. In the literature, we identify three different approaches of coincidence concept to compute soft consensus measure:

1. *Consensus models based on strict coincidence among preferences.* In this case, similarity criteria among preferences are used to compute the coincidence concept. It is assumed only two possible results: the total coincidence (value 1) or null coincidence (value 0). Some examples of this approach are the following: In [19], assuming fuzzy preference relations to represent experts' preferences, the first consensus model based on strict coincidence was defined. Given a particular alternative pair and two experts, if their preferences are equal, then they are in agreement (value 1), and otherwise they

are in disagreement (value 0). Then consensus measures are calculated across the global set of the alternatives in a hierarchical pooling process from the coincidence measured on experts' preferences and using the fuzzy majority concept represented by a linguistic quantifier [29]. In [9,10], different consensus measures based on strict coincidence were presented assuming that experts' preferences are provided by means of linguistic preference relations. Applying the strict coincidence on preferences provided by the experts for each alternative pair, the expert group is divided into subsets, one subset for each possible linguistic label used to quality the preference on the alternative pair. Then, using the cardinalities of the subsets of experts three kinds of consensus measures are defined, each one associated to the three different levels of representation of a preference relation, alternative pair, individual alternative and global relation.

2. *Consensus models based on soft coincidence among preferences.* As above, similarity criteria among preferences are used to compute the coincidence concept but, in this case, a major number of possible coincidence degrees is considered. It is assumed that the coincidence concept is a gradual concept which could be assessed with different degrees defined in the unit interval [0,1]. Some examples of this approach are the following: In [19], a first consensus model based on soft coincidence was also defined. But in this case, given a particular alternative pair and two experts, the coincidence among their preference is measured using a closeness function $s : [0,1] \rightarrow [0,1]$. In [20,21], some soft consensus measures defined as extensions of those shown in [19] are introduced, considering GDM problems with heterogeneous set of alternatives and heterogeneous groups of experts, respectively. In [7], an extension of soft consensus models defined in [19,20,21] is presented, which consists in the computation of consensus measures using the ordered weighted averaging (OWA) operator [28]. In [4], a soft consensus model for multi-criteria GDM problems defined in a ordinal fuzzy linguistic approach was defined. In this case, coincidence values are obtained by means of a linguistic similarity function defined directly on linguistic assessments given on the alternatives. In [11], the fuzzification of soft coincidence concept was presented. The soft coincidence is defined in each alternative pair of a linguistic preference relation as a fuzzy set defined on the set of expert pairs and characterized by closeness observed among their preferences. The closeness among preferences is established by means of ad-hoc closeness table defined among all the possible labels of linguistic term set used to represent the preferences. In [14], a soft consensus model is presented to deal with GDM problems in a multi-granular fuzzy linguistic context. As in [9,10,11], three kinds of soft consensus measures are considered. The soft coincidence among multi-granular linguistic preferences is obtained using a similarity function defined on transformation of such preferences in a basic linguistic term set. In [16], as in [9,10,11,14], a soft consensus model based on three consensus measures was proposed. In this case, experts provide their preferences by means of incomplete fuzzy preference relations assessed in [0,1] and the soft coincidence is defined using a similarity function among preferences in [0,1].

3. *Consensus models based on coincidence among on solutions.* In this case, similarity criteria among the solutions obtained from the experts' preferences are used to compute the coincidence concept and different degrees assessed in [0,1] are assumed [1,13]. Basically, we compare the positions of the alternatives between the individual solutions and the collective solution, which allows to know better the real consensus situation in each moment of the consensus process. Some examples of this approach are the following: In [13] was defined the first consensus model based on the measuring the coincidence degree between individual solutions and collective solution. In [13], it is assumed that experts represent their preferences by means of different elements of representation (relation, ordering and utilities) and then it is not possible to compare preferences. To overcome this problem authors propose to compare solutions to obtain the coincidence degrees. This means that the first step of consensus process to measure coincidence degrees is to apply a selection process to obtain a temporary collective solution and the temporary individual solutions, and measure the closeness among them. An important characteristic of this consensus model was the introduction of a recommendation system to aid experts to change their preferences in the consensus reaching process and, in such a way, to substitute the moderator's actions. In [1], a similar consensus model is presented but assuming heterogeneous GDM problems, i.e., experts with different importance degrees.

3 Discussion

In this section, some important aspects of the use of the different approaches to obtain soft consensus degrees within the decision making process are analyzed. To do so, we show the advantages and drawbacks of each one of them.

1. *Strict coincidence among preferences.* The advantage of this approach is that the computation of the consensus degrees is simple and easy because it assumed only two possible values: 1 if the opinions are equal and otherwise a value of 0. However, the drawback of this approach is that the consensus degrees obtained do not reflect the real consensus situation because it only assigns values of 1 or 0 when comparing the experts' opinions, and, for example, we obtain a consensus value 0 for two different preference situations as (very high, high) and (very high,low), when clearly in the second case the consensus value should be lower than in the first case.
2. *Soft coincidence among preferences.* The advantage of this approach is that the consensus degrees obtained are similar to the real consensus situation because they are obtained using similarity functions that assign values between 0 and 1, which are not so strict as in the above approach. The drawback of this approach is that the computation of the consensus degrees is more difficult than in the above approach because we need to define similarity criteria [14,16].
3. *Coincidence among solutions.* The advantage of this approach is that the consensus degrees are obtained comparing not the opinions or choice de-

grees but the position of the alternatives in each solution, what allows us to reflect the real consensus situation in each moment of the consensus reaching process. The drawback of this approach is that the computation of the consensus degrees is more difficult than in the above approaches because we need to define similarity criteria and it is necessary to apply a selection process before obtaining the consensus degrees.

4 New Advanced Approaches

In this section, we describe the new advanced soft consensus approaches which have been developed using the above concepts of coincidence. These approaches allow to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible [13,14,16] and adapt the consensus process to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round [25].

4.1 Approaches Generating Recommendations to Help Experts

These approaches generate simple and easy rules to help experts change their opinions in order to obtain the highest degree of consensus possible. To do so, they are based on two consensus criteria, consensus degrees indicating the agreement between experts opinions and proximity measures used to find out how far the individual opinions are from the group opinion. Thus, proximity measures are used in conjunction with the consensus degrees to build a guidance advice system, which acts as a feedback mechanism that generates advice so that experts can change their opinions. Furthermore, these consensus criteria are computed at the three different levels of representation of information of a preference relation: pair of alternatives, alternative, and relation. It allows us to know the current state of consensus from different viewpoints, and therefore, to guide more correctly the consensus reaching processes. Thus, as these measures are given on three different levels for a preference relation, this measure structure will allow us to find out the consensus state of the process at different levels. For example, we will be able to identify which experts are close to the consensus solution, or in which alternatives the experts are having more trouble to reach consensus.

So, the computation of the consensus degrees assuming that experts provide their preferences by means of fuzzy preference relations, $P^h = (p^h_{ij})$, is carried out as follows. First, for each pair of experts (e_h, e_l) $(h = 1, \ldots, m-1,\ l = h+1, \ldots, m)$ a similarity matrix $SM^{hl} = (sm^{hl}_{ik})$ is defined. To do it, one of the above coincidence criteria can be used. Then, a collective similarity matrix, $SM = (sm_{ik})$, is obtained by aggregating all the similarity matrices using an aggregation function ϕ

$$sm_{ik} = \phi(sm^{hl}_{ik},\ h = 1, \ldots, m-1,\ l = h+1, \ldots, m). \tag{1}$$

Once the similarity matrices are computed, the consensus degrees are calculated at the three different levels.

1. **Level 1.** *Consensus degree on pairs of alternatives.* The consensus degree on a pair of alternatives (x_i, x_k), called cop_{ik}, is defined to measure the consensus degree amongst all the experts on that pair of alternatives. In this case, this is expressed by the element of the collective similarity matrix SM, i.e.,
$$cop_{ik} = sm_{ik}. \tag{2}$$
The closer cop_{ik} to 1, the greater the agreement amongst all the experts on the pair of alternatives (x_i, x_k). This measure will allow the identification of those pairs of alternatives with a poor level of consensus.
2. **Level 2.** *Consensus degree on alternatives.* The consensus degree on alternative x_i, denoted ca_i, is defined to measure the consensus degree among all the experts on that alternative:
$$ca_i = \frac{\sum_{k=1;k\neq i}^{n} (cop_{ik} + cop_{ki})}{2n-2}. \tag{3}$$
These values can be used to propose the modification of preferences associated to those alternatives with a consensus degree lower than a minimal consensus threshold γ.
3. **Level 3.** *Consensus degree on the relation.* The consensus degree on the relation, called cr is defined to measure the global consensus degree amongst all the experts' opinions. It is computed as the average of all the consensus degrees on the alternatives, i.e.,
$$cr = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{4}$$
This is the value used to control the consensus situation.

Once consensus degrees are calculated, the proximity measures are obtained. To compute them for each expert, we need to obtain the collective preference relation, $P^c = (p_{ik}^c)$, which summarizes preferences given by all the experts and is calculated by means of the aggregation of the set of individual preference relations $\{P^1, \ldots, P^m\}$ as follows
$$p_{ik}^c = \phi(p_{ik}^1, \ldots, p_{ik}^m). \tag{5}$$
with ϕ an aggregation operator.

Once P^c is obtain, we can compute the proximity measures carrying out the following two steps:

1. For each expert, e_h, a proximity matrix, $PM^h = (pm_{ik}^h)$, is obtained using one of the above coincidence criteria.
2. Computation of proximity measures at three different level:
   (a) **Level 1.** *Proximity measure on pairs of alternatives.* The proximity measure of an expert e_h on a pair of alternatives (x_i, x_k) to the group's one, called pp_{ik}^h, is expressed by the element (i, k) of the proximity matrix PM^h:
$$pp_{ik}^h = pm_{ik}^h. \tag{6}$$

(b) **Level 2.** *Proximity measure on alternatives.* The proximity measure of an expert e_h on an alternative x_i to the group's one, called pa_i^h, is calculated as follows:

$$pa_i^h = \frac{\sum_{k=1,k\neq i}^{n} pp_{ik}^h}{n-1}. \tag{7}$$

(c) **Level 3.** *Proximity measure on the relation.* The proximity measure of an expert e_h on his/her unbalanced fuzzy linguistic preference relation to the group's one, called pr^h, is calculated as the average of all proximity measures on the alternatives:

$$pr^h = \frac{\sum_{i=1}^{n} pa_i^h}{n}. \tag{8}$$

The meaning of the proximity measures are the following: if they are close to 1, then they have a positive contribution for the consensus to be high, while if they are close to 0, then they have a negative contribution to the consensus. Therefore, we can use them to provide advice to the experts to change their opinions and to find out which direction that change has to follow in order to obtain the highest degree of consensus possible.

Once proximity measures are calculated, the recommendations are generated. The production of advice to achieve a solution with the highest degree of consensus possible is carried out in two steps [14]: *Identification rules* and *Direction rules.*

1. **Identification rules (IR).** We must identify the experts, alternatives and pairs of alternatives that are contributing less to reach a high degree of consensus and, therefore, should participate in the change process.

   (a) *Identification rule of experts (IR.1).* It identifies the set of experts that should receive advice on how to change some of their preference values. This set of experts, called $EXPCH$, that should change their opinions are those whose satisfaction degree on the relation is lower than the minimum consensus threshold γ. Therefore, the identification rule of experts, IR.1, is the following:

   $$EXPCH = \{e_h \mid pr^h < \gamma\} \tag{9}$$

   (b) *Identification rule of alternatives (IR.2).* It identifies the alternatives whose associated assessments should be taken into account by the above experts in the change process of their preferences. This set of alternatives is denoted as ALT. The identification rule of alternatives, IR.2, is the following:

   $$ALT = \{x_i \in X \mid ca_i < \gamma\} \tag{10}$$

   (c) *Identification rule of pairs of alternatives (IR.3).* It identifies the particular pairs of alternatives (x_i, x_k) whose respective associated assessments

p_{ik}^h the expert e_h should change. This set of pairs of alternatives is denoted as $PALT^h$. The identification rule of pairs of alternatives, IR.3, is the following:

$$PALT^h = \{(x_i, x_k) \mid x_i \in ALT \wedge e_h \in EXPCH \wedge pp_{ik}^h < \gamma\} \tag{11}$$

2. **Direction rules (DR).** We must find out the direction of the change to be recommended in each case, i.e., the direction of change to be applied to the preference assessment p_{ik}^h, with $(x_i, x_k) \in PALT^h$. To do this, we define the following four direction rules.
   (a) *DR.1.* If $p_{ik}^h > p_{ik}^c$, the expert e_h should decrease the assessment associated to the pair of alternatives (x_i, x_k), i.e., p_{ik}^h.
   (b) *DR.2.* If $p_{ik}^h < p_{ik}^c$, the expert e_h should increase the assessment associated to the pair of alternatives (x_i, x_k), i.e., p_{ik}^h.

4.2 Adaptive Approaches

These approaches are based on a refinement process of the consensus process that allows to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round. The refinement process adapts the search for the furthest experts' preferences to the existent agreement in each round of consensus. So, when the agreement is very low (initial rounds of the consensus process), the number of changes of preferences should be bigger than when the agreement is medium or high (final rounds) (see Figure 2).

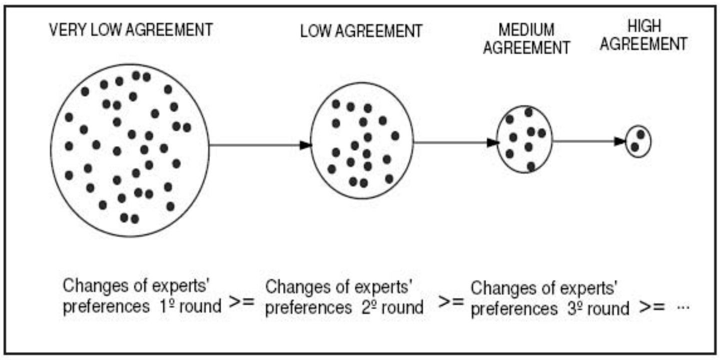

Fig. 2. Reduction of the number of changes of preferences into the consensus process

These approaches consider that in the first rounds of the consensus process, the agreement is usually very low and it seems logic that many experts' preferences should be changed. However, after several rounds, the agreement should have improved and then just the furthest experts' preferences from the collective preference should be changed. It involves that the procedure to search for the furthest experts' preferences from collective preference should be different according to the achieved agreement in each round. Each Preference Search Procedure (PSp) should have a different behavior and should return a different set of preferences that each expert should change in order to improve the agreement in the next consensus round. In consequence of the adaptation of the consensus process to the existent agreement in each round, the number of changes of preferences suggested to experts after each consensus round will be smaller according to the favorable evolution of the level of agreement.

In this way, in the consensus process, if the agreement among experts is low, i.e, there are a lot of experts' preferences with different assessments, the number of experts which should change their preferences in order to make them closer to collective preference should be great. However, if the agreement is medium or high, it means that the majority of preferences are similar and therefore there exist a low number of experts' preferences far from the collective preference. In this case, only these experts should change them in order to improve the agreement. Keeping in mind this idea, these approaches propose distinguishing among three level of agreement: very low, low and medium consensus. Each level of consensus involves to carry out the search for the furthest preferences in a different way. So when the consensus degree cr is very low, these approaches will search for the furthest preferences on all experts, while if cr is medium, the search will be limited to the furthest experts. To do so, these approaches carries out three different PSps: PSp for very low consensus, PSp for low consensus and PSp for medium consensus. The possibility of carrying out different PSps according to the existent consensus degree in each round defines the adaptive character of our model.

5 Concluding Remarks

In this paper we have identified the different existing approaches to compute soft consensus measures in fuzzy group decision making problems and analyzed their advantages and drawbacks. Additionally, we have described the new advanced approaches allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible and adapt the consensus process to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round.

In the future, we think to study as to apply these consensus models in decision making problems with incomplete information and using information domains which do not allow to define similarity criteria among preferences in a direct way, as for example the unbalanced fuzzy linguistic information [12,17].

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